NATIONAL SENIOR CERTIFICATE EXAMINATION
MAY 2023

MATHEMATICS: PAPER I

MARKING GUIDELINES

Time: 3 hours | 150 marks

These marking guidelines are prepared for use by examiners and sub-examiners, all of whom are required to attend a standardisation meeting to ensure that the guidelines are consistently interpreted and applied in the marking of candidates' scripts.

The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines. It is also recognised that, without the benefit of attendance at a standardisation meeting, there may be different interpretations of the application of the marking guidelines.

SECTION A

QUESTION 1

(a) $x = 0$ or $x = 5$ or $x = \frac{1}{3}$

(b) $\sqrt{2x-1} - 7 = 0$

$2x - 1 = 49$

$x = 25$

(c) $y = 2x - 5$

$x^2 - (2x - 5)^2 = 7$

$x^2 - 4x^2 + 20x - 25 = 7$

$3x^2 - 20x + 32 = 0$

$(3x - 8)(x - 4) = 0$

$x = 4$ or $x = \frac{8}{3}$

$y = 3$ or $y = \frac{1}{3}$

QUESTION 2

(a) (1) $f'(x) = \lim_{h \to 0} \frac{2(x+h)^2 + 5 - (2x^2 + 5)}{h}$

$= \lim_{h \to 0} \frac{2(x^2 + 2xh + h^2) + 5 - (2x^2 - 5)}{h}$

$= \lim_{h \to 0} \frac{2x^2 + 4xh + 2h^2 + 5 - 2x^2 - 5}{h}$

$= \lim_{h \to 0} \frac{4xh + 2h^2}{h}$

$= \lim_{h \to 0} \frac{h(4x + 2h)}{h}$

$f'(x) = 4x$ (Notation)

(a) (2) $f'(3) = 4(3) = 12$

$f(3) = 2(3)^2 + 5 = 23$

$y = 12x + c$

$23 = 12(3) + c$

$c = -13$

Tangent equation $y = 12x - 13$

Alternate Solution: $y - 23 = 12(x - 3)$

(b) $g(x) = \sqrt{x} + x + \frac{1}{x}$

$g(x) = x^{\frac{1}{2}} + x + x^{-1}$

$g'^{(x)} = \frac{1}{2}x^{-\frac{1}{2}} + 1 - x^{-2}$

QUESTION 3

(a) (1) 29 dots

(2) $Tn = (n + 1)^2 + n$

$Tn = n^2 + 3n + 1$

Alternate solution

5; 11; 19; 29......

$2a = 2$ $3(1) + b = 6$ $(1) + (3) + c = 5$

$Tn = n^2 + 3n + 1$

(b) $T_4 = 29$

$T_{14} = 44$

$d = \frac{15}{10} = 1{,}5$

$T_1 = 29 - 3(1{,}5) = 24{,}5$

Alternate Solution

$a + 3d = 29$

$a + 13d = 44$

$\therefore\ 10d = 15$

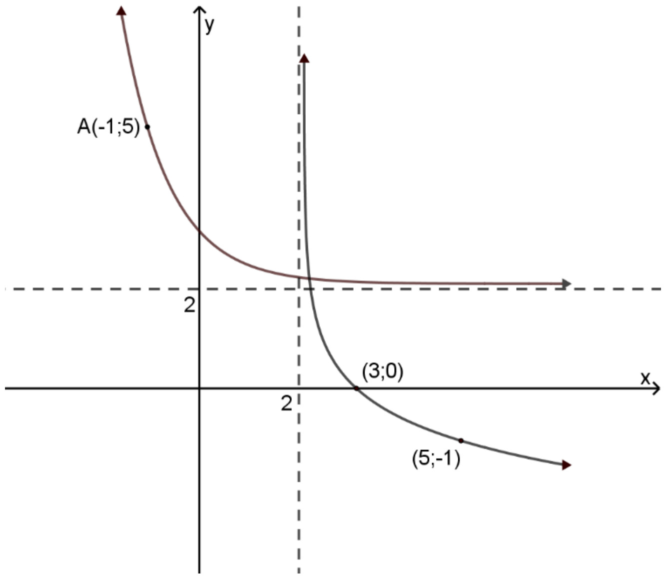
d = 1,5

QUESTION 4

(a) (1) $a = \frac{1}{3}$ and $q = 2$

(2) y intercept of g is $(0; 3)$

(3)



x intercept point (5; –1) Assymptote

(4) $2 < x < 3$

(b) (1) $f(x) = -x^2 + 5x - 4$

$C(0; -4)$

$0 = -x^2 + 5x - 4$

$x^2 - 5x + 4 = 0$

$x = 4$ or $x = 1$

$A(1; 0)$ and $B(4; 0)$

$f'(x) = -2x + 5$

$-2x + 5 = 0$

$x = \frac{5}{2}$

Turning point of $f(x)$ is $D\left(\frac{5}{2}; \frac{9}{4}\right)$

(2) $t = -\frac{9}{4}$

QUESTION 5

(a) The horizontal asymptote

$y = (-1) + 3$

$y = 2$

(b1) CD = 2 units

(b2) $\frac{3}{x+1} + 2 = x + 3$

$\frac{3}{x+1} = x + 1$ or $3 + 2(x + 1) = (x + 3)(x + 1) \therefore x^2 + 2x - 2 = 0$

$3 = (x + 1)^2$

$x = -1 \pm \sqrt{3}$

$A\left(-1+\sqrt{3}; 2+\sqrt{3}\right)$

$B\left(-1-\sqrt{3}; 2-\sqrt{3}\right)$

(c) $x \in [-2{,}7; -1) \cup [0{,}7; \infty)$

Or

$x \in [-1-\sqrt{3}; -1) \cup [-1+\sqrt{3}; \infty)$

QUESTION 6

(a) $$820\,000 = \frac{x\left[1-\left(1+\frac{0,10}{12}\right)^{-240}\right]}{\frac{0,10}{12}}$$

x = R7 913,18

(b) $$820\,000 = \frac{20\,000\left[1-\left(1+\frac{0,10}{12}\right)^{-n}\right]}{\frac{0,10}{12}}$$

$\left(1+\frac{0,10}{12}\right)^{-n} = 0,6583333333 \therefore\ n = 50,4$, hence 51 months.

SECTION B

QUESTION 7

$$F_v = \frac{750\left[\left(1+\frac{0,15}{12}\right)^{36} - 1\right]}{\frac{0,15}{12}} \cdot \left(1+\frac{0,15}{12}\right)^{84} = \text{R}96\ 066,\ 01366$$

$$F_v = \frac{500\left[\left(1+\frac{0,15}{12}\right)^{24} - 1\right]}{\frac{0,15}{12}} \cdot \left(1+\frac{0,15}{12}\right)^{60} = \text{R}29\ 277,\ 26617$$

At the end of 10 years the investment is worth R125 343, 28

QUESTION 8

(a) $3^{x+1} + 3^{x-1} = 20$

$3^x(3 + 3^{-1}) = 20$

$3^x = 6$

$x = \log_3 6$

Therefore

$w = 3$ and $t = 6$

(b) $\frac{5^{-1}75^{2x}}{15^{4x}3^{-3x}} = k \times 3^x$

$\frac{5^{-1}3^{2x}5^{4x}}{3^{4x}5^{4x}3^{-3x}}$

$5^{-1}.3^x$

Therefore

$k = \frac{1}{5}$

(c) $3^{x^2 - px + 1} = 27$

$x^2 - px + 1 = 3$

$x^2 - px - 2 = 0$

$\Delta = (-p)^2 - 4(1)(-2)$

$\Delta = p^2 + 8$

Therefore $\Delta > 0$ and hence the roots are always real and unequal.

QUESTION 9

$y = a(x - 100)^2 + q$

Sub in (200;30)

$30 = a(200 - 100)^2 + q$

$30 = 10\ 000a + q$

Sub in (300;0)

$0 = a(300 - 100)^2 + q$

$0 = 40\ 000a + q$

$q = -40\ 000a$

Sub into first equation

$30 = 10\ 000a - 40\ 000a$

$30 = -30\ 000a$

$a = \frac{-1}{1\ 000}$

$q = 40$

The golf ball reaches a maximum height of 40 metres.

Alternate: $y = ax^2 + bx + 30$

substitute (200;30) & (300;0)

$30 = 40\ 000a + 200b + 30$ & $0 = 90\ 000a + 300b + 30$

$a = \frac{-1}{1\ 000}$ & $b = \frac{1}{5}$ hence $y = \frac{-1}{1\ 000}x^2 + \frac{1}{5}x + 30$

$\frac{dy}{dx} = 0$

Thus $\frac{-2x}{1\ 000} + \frac{1}{5} = 0 \quad \therefore\ x = 100$ and $y = 40$

QUESTION 10

(a) $0 = -x^3 + x^2 + 21x - 45$

$0 = x^3 - x^2 - 21x + 45$

$0 = (x - 3)(x^2 + 2x - 15)$

$0 = (x - 3)(x - 3)(x + 5)$

$x = 3$ or $x = -5$

(b) $h(x) = x^3 + ax^2 + bx + 36$

$h'(x) = 3x^2 + 2ax + b$

$B(-1; 44)$

$h(-1) = -1 + a - b + 36$

$-1 + a - b + 36 = 44$

$a - b = 9$

$h'(x) = 3x^2 + 2ax + b$

$h'(-1) = 3 - 2a + b$

$3 - 2a + b = 0$

$b = 2a - 3$

Solve simultaneously:

$a - (2a - 3) = 9$

$a - 2a + 3 = 9$

$a = -6$

$b = 2(-6) - 3$

$b = -15$

QUESTION 11

(a) $26x + 10y = 10$

$y = 1 - 2{,}6x$

$V = 4x^2y$

$V = 4x^2(1 - 2{,}6x)$

$V = 4x^2 - 10{,}4x^3$

(b) $\frac{dV}{dx} = 8x - 31{,}2x^2$

$8x - 31{,}2x^2 = 0$

$x(8 - 31{,}2x) = 0$

$x \neq 0$ or $x = 0{,}2564$

$y = 0{,}33$

QUESTION 12

(a) $\frac{605}{243} = \frac{w(r^5 - 1)}{r - 1}$

$T_5 = w.3^{-5}$ therefore $r = \frac{1}{3}$ or $T_1 = \frac{w}{3}$ and $T_2 = \frac{w}{9}$ thus $r = \frac{1}{3}$

$$\frac{605}{243} = \frac{w\left(\left(\frac{1}{3}\right)^5 - 1\right)}{\left(\frac{1}{3}\right) - 1}$$

$w = \frac{5}{3}$

(b) $\sum_{n=1}^{\infty}(2^{-n}) + \sum_{n=1}^{p}(2n + 1) = 484$

$$S\infty = \frac{\frac{1}{2}}{1 - \frac{1}{2}} = 1$$

$\sum_{n=1}^{p}(2n + 1) = 483$

3 + 5 + 7 + 9 + ……. + (2p + 1) = 483

$483 = \frac{p}{2}(2(3) + (p - 1)(2))$

$483 = \frac{p}{2}(2p + 4)$

$0 = p^2 + 2p - 483$

$0 = (p + 23)(p - 21)$

$p \neq -23$ or $p = 21$

QUESTION 13

(a) (1) $\dfrac{8!}{3!2!} = 3\,360$

(2) 8 _ _ _ _ _ _ 5 $\dfrac{6!}{2!}$

Probability $= \dfrac{360}{3\,360} = \dfrac{3}{28} = 0{,}11$

(3) $\dfrac{9!}{3!2!x} = 15\,120$

$x = 2!$

Therefore

3,4 or the 7 could be added.

(b)

		Red Die					
		1	2	3	4	5	6
Green Die	1		2	3	4	5	6
	2	2		6	8	10	12
	3	3	6		12	15	18
	4	4	8	12		20	24
	5	5	10	15	20		30
	6	6	12	18	24	30	

Red	Green	Perimeter < 22
3	5	Y
3	6	Y
4	5	Y
4	6	Y
5	3	Y
5	4	Y
5	6	N
6	3	Y
6	4	Y
6	5	N

Area workings Perimeter workings

P(Are > 12; Perimeter < 22 and not a square) $= \dfrac{8}{36} = 0{,}22$

Total: 150 marks